Effect of Damping and Gain Compression in Purcell-Enhanced Nanocavity Lasers

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Abstract: We simulate the rate equations of nanocavity lasers, introducing gain compression of both the stimulated and spontaneous emission. The resonance frequency and damping are simultaneously enhanced by the Purcell effect, greatly limiting the modulation bandwidth.

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1. Introduction

Semiconductor nanocavities hold great promise as thresholdless lasers for increased power efficiency and also as platforms for observing quantum electrodynamic phenomenon. Following the theoretical prediction that the Purcell-enhanced spontaneous emission rate could significantly increase modulation speeds [1], several papers have been published on both theoretical and experimental investigations of Purcell-enhanced bandwidth [2]. However, all the papers to date have neglected the damping effect due to gain compression. In this paper, we will show that gain compression is the *dominant* factor limiting the bandwidth of Purcell-enhanced nanocavity lasers. We analyze the modulation speed of nanocavity lasers using the laser rate equation approach. In a nanocavity laser, the enhanced spontaneous emission contributes a significant portion of the laser power. Einstein showed that the spontaneous emission into a cavity mode is proportional to the laser gain via stimulated emission. Therefore, the spontaneous emission must also be subject to gain compression effects. Here, we show that the Purcell-enhanced resonance frequency is dramatically reduced by gain compression of the spontaneous emission, much in the same way as a conventional laser's resonance frequency is reduced by gain compression of the stimulated emission [3]. We find that this ultimately limits the laser bandwidth to frequencies well below the relaxation oscillation frequency.

2. Rate Equations of Purcell-Enhanced Nanocavity Lasers

To describe the dynamics of nanocavity lasers, the classic laser rate equations are modified to include the Purcell factor, *F*. Here, we take the nanocavity rate equations of [4] and spatially integrate them to obtain a lumped-element model:

$$\frac{dN}{dt} = J - GS - [\beta F + (1-\beta)]BN^2 - CN^3 \tag{1a}$$

$$\frac{dS}{dt} = \left[\Gamma G - \frac{1}{\tau_p}\right] S + \Gamma \beta F B N^2 , \tag{1b}$$

where N is the carrier density, S is the photon density, J is the carrier injection rate, G is the laser gain, β is the spontaneous emission factor, B and C are the bimolecular and Auger recombination coefficients, respectively, Γ is the confinement factor, and τ_P is the photon lifetime. The spontaneous emission rate into the lasing mode is enhanced by the Purcell factor [5], $F = 2Q/\pi^2 V_n$, where $V_n = V/(\lambda/2n)^3$ is the normalized modal volume, λ is the lasing wavelength, and n is the effective index of the laser mode. We define β as the ratio of spontaneous emission rate into the lasing mode, $R_{sp}^{'}$, versus the total spontaneous emission rate, R_{sp} [6]:

$$\beta = \frac{R_{sp}^{'}}{R_{sp}} = \frac{G n_{sp}}{VBN^2} \tag{2}$$

where n_{sp} is the population inversion factor and V is the modal volume. The spontaneous emission rate is proportional to the stimulated emission rate, thus is subject to gain compression, through the gain, G. We use a quantum-well approximation for the population inversion factor [7] and a logarithmic gain model with gain compression, giving $G = g_0 \ln(N/N_{tr})(1+\varepsilon S)^{-1}$, where g_0 is the gain coefficient, N_{tr} is the carrier density at transparency, and ε is the gain compression factor.

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3. Simulation Results

There are two independent parameters for nanocavity lasers: the cavity quality factor, Q, and the normalized modal volume, V_n. We performed a small-signal analysis on the rate equations in the parameter space of Q vs. V_n. For each Q-V_n pair, we solved the DC operating condition that will result in maximum 3-dB frequency, $f_{3dB,max}$ (by scanning carrier injection rate). Fig. 1(a) shows the contours of $f_{3dB,max}$ in the Q vs. V_n plane. The Purcell effect significantly affects the modulation dynamics for volumes left of the dotted line. For a conventional laser with $V_n = 1000$, reducing Q results in higher bandwidths. However, further reduction of Q below ~1000 results in higher threshold carrier densities, lower differential gain, and smaller maximum bandwidths. As the cavity volume reduces and transitions into the Purcell-enhanced regime, the bandwidth only begins to enhance for $V_n < 0.5$; well below the volume where Purcell-enhanced dynamics become significant. This is because the Purcell-enhanced resonance frequency is severely damped by the Purcell-enhanced damping factor. In Fig. 1(b), we show the frequency response of a particular example: $Q = 1000$ and $V_n = 0.5$, as marked on Fig. 1(a). Although the resonance frequency, f_R, is 117 GHz, the damping factor, γ, is 293 GHz, limiting the $f_{3dB,max}$ to 55 GHz, which actually has slightly smaller bandwidth than the non-Purcell-enhanced case with equivalent Q ($V_n = 1000$), also shown in Fig. 1(b). In this classical case, f_R, γ, and f_{3dB} = 50, 60, and 57 GHz, respectively. Fig. 1(c) compares the f_R, γ, and $f_{3dB,max}$ for different volumes and a fixed $Q = 1000$. Although the resonance frequency begins increasing for $V_n < 10$, the simultaneously increasing damping factor suppresses the bandwidth. Only at extremely small cavities ($V_n < 0.5$) do we begin to see appreciable bandwidth enhancement. It is interesting to note that the bandwidth has a stronger correlation to the inverse product $1/QV_n$, rather than simply the Purcell factor, which is proportional to the ratio Q/V_n.

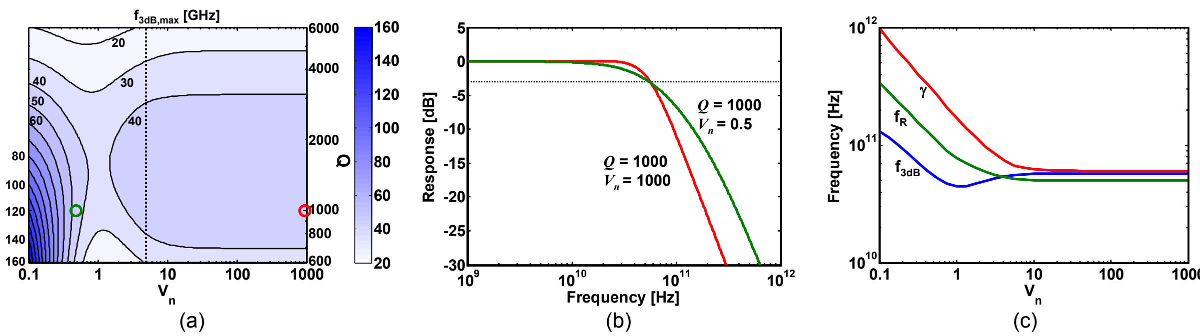

Fig. 1. (a) Maximum 3-dB frequency as a function of cavity-Q and normalized modal volume, $V_n = V/(\lambda/2n)^3$. Left and right regions, separated by the dotted line, are the Purcell-enhanced and classical laser regimes, respectively. Circles mark the example cases shown in (b): frequency response curves comparing a Purcell-enhanced ($V_n = 0.5$) and non-Purcell enhanced ($V_n = 1000$) cavity at the same $Q = 1000$. (c) Comparison of f_{3dB}, f_R, and γ for $Q = 1000$.

It should be noted that damping from both stimulated and Purcell-enhanced spontaneous emission contribute to the overall damping of the laser; gain compression enhances both. Without the addition of the gain compression on the spontaneous emission factor, the bandwidth will saturate at much higher frequencies, much in the same way gain compression limits the maximum bandwidth of a conventional laser [3]. For a nanocavity laser with $V_n = 0.1$ and $Q = 1000$, without gain compression, $f_{3dB,max}$ would be 346 GHz. With gain compression only considered on the conventional stimulated emission term, $f_{3dB,max}$ would be 227 GHz. Both of these overestimate the more realistic $f_{3dB,max}$ = 167 GHz bandwidth that also takes gain compression on the spontaneous emission factor into account.

In conclusion, we have shown that gain compression on the spontaneous emission factor dramatically reduces the usable 3-dB frequency of Purcell-enhanced laser. Our results also show that for high-bandwidth nanocavity lasers, volumes of $V_n < 0.5$ must be chosen, with $Q < 2000$. Extremely low Q nanocavity lasers are feasible since the Purcell effect can reduce threshold pumping levels dramatically.

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